CHARACTERISTICS OF PRESSURE IN MATHEMATICAL MODELING OF THE HYDRODYNAMIC PROCESS OF IN-SITU LEACHING TAKING INTO ACCOUNT THE CHANGE OF HYDRODYNAMIC PARAMETERS IN THE PORE ENVIRONMENT

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ABSTRACT

This study is devoted to mathematical modeling of In-Situ leaching (ISL) and reagent-environment interactions in solution mining technology. The effectiveness of the ISL process directly depends on the correct description of the hydrodynamics, reactive transport and interphase mass transfer processes that occur within pore and fractured geologic domains.. In this work, these interrelated processes were brought to rigorous mathematical expressions based on the theory of multi-component and multi-phase reactive flows. In forming the model idea, the classical Darcy law for the pressure field, dispersion-diffusion equations for mass transport, as well as kinetic models describing the kinetics of interphase reactions were used. Computational experiments made it possible to analyze in depth the influence of parameters such as permeability anisotropy of geological layers, porosity coefficient, hydrostatic gradients, initial reagent concentration and current density on the mixing process. The results revealed fundamental laws regarding the geometry of the development of the reagent front, the radial and vertical evolution of the melting zone, the transformation of the chemical composition over time, and the collective strength of useful components. In particular, it was found that the increase in dispersion and the formation of local reactive zones in porous systems significantly modify the overall dynamics of the process. The results of the research provide a scientific basis for solving important engineering issues, such as optimal well location, reagent consumption minimization, jet transport safety limits, and ensuring stable operation of the field for the ISL technology. The proposed mathematical model is structurally universal and adaptable to various

geochemical conditions; it serves as a robust platform for integration with real-time monitoring data and optimization of management strategies.

In view of the above, the current paper proposes a mathematical model to solve pressure and kinetic related problems in an ore deposit.

Physical statement of the problem

We consider a specialized field for mathematical modeling of the ISL process.

$G=\{(x,y,z,t),\ 0<x<L_x,\ 0<y<L_y,\ 0<z<L_z,\ 0<t\quad T\}.$

The distribution of the pressure field is determined from the equations of the elastic filtering process, as in [1-3].

$$\beta h\frac{\partial(mH)}{\partial t}=\frac{\partial}{\partial x}\ \kappa h\frac{\partial H}{\partial x}\ +\frac{\partial}{\partial y}\ \kappa h\frac{\partial H}{\partial y}\ +\frac{\partial}{\partial z}\ \kappa h\frac{\partial H}{\partial z}\ +F_1-F_2, \quad (1)$$

$$m=m_0+\beta_c(H-H_0) \quad (2)$$

initial:

$$H\big|_{t=0}=H_0, \quad (3)$$

and boundary conditions:

$$\kappa\frac{\partial H}{\partial x}\bigg|_{x=0}=-\iota_1\xi(H-H_0), \quad (4)$$

$$\kappa\frac{\partial H}{\partial x}\bigg|_{x=L_x}=\iota_1\xi(H-H_0), \quad (5)$$

$$\kappa\frac{\partial H}{\partial y}\bigg|_{y=0}=-\iota_2\xi(H-H_0), \quad (6)$$

$$\kappa\frac{\partial H}{\partial y}\bigg|_{y=L_y}=\iota_2\xi(H-H_0), \quad (7)$$

$$\frac{\partial H}{\partial z}\bigg|_{z=0}=0, \quad (8)$$

$$\kappa\frac{\partial H}{\partial z}\bigg|_{z=L_z}=\iota_3\xi(H-H_0), \quad (9)$$

The diffusion of the reactive field is determined by solving the convective diffusion equation. The desired distribution of the concentration function of the useful component is determined by solving the following equation. The formulation of these equations and the solution of their corresponding initial and boundary value problems are presented in detail in the article [1].

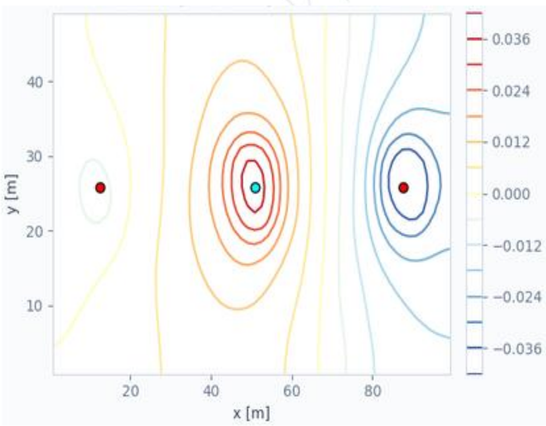
Here: H–the pressur, (m); H_0–the initial pressur, (m); m–porosity coefficien; k–filtration coefficient, (m/ day); t–time (day). h–thickness of the ore horizon (m); β–coefficient of elastic capacity, (m^2/kg); $\iota_1,\iota_2,\iota_3,$ – constants that take the values 0 or 1; L_x – length of field with respect to Ox axis (m); L_y – length of field with respect to Oy axis (m); L_z – length of field with respect to Oz axis (m); ξ –dimensionless coefficient of equation (1/day); $F_1=\sum_{i,j=1}^{N_1} q_{1,i}(t)\delta\left(x-x_{i,j},y-y_{i,j},z-z_{i,j}\right)$ –power of the injection wells;

$F_2 = \sum_{i,j=1}^{N_2} q_{2,j}(t)\delta(x - x_{i,j}, y - y_{i,j}, z - z_{i,j})$ –power of the production wells; $q_{i,j}(t)$ –flow rate of the wells; N_1 and N_2 are the number of the injection and production wells, respectively; $\delta_{i,j} = \begin{cases} 1, x = x_{i,j}, y = y_{i,j}, z = z_{i,j} \\ 0, \text{other condation} \end{cases}$ – is Dirac's delta function .

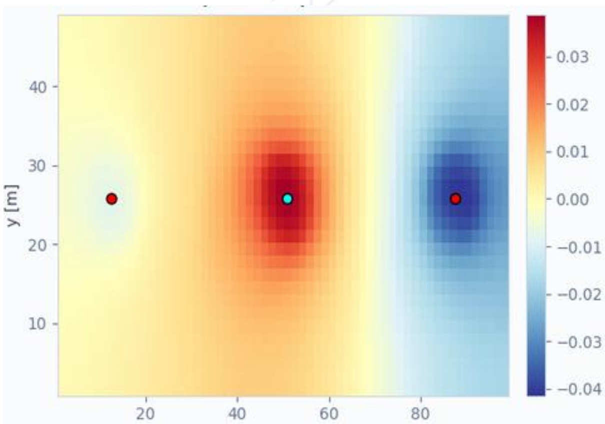
From the statement of the problem, it can be seen that it is expressed by a system of multidimensional equations and is difficult to obtain an analytical solution. Taking this into account, the problem of replacing the learned domain with a discrite domain, replacing the differential operators in the equations with special operators using the finite element method, and computational experiments were conducted on a computer in [1] article.

Results analysis

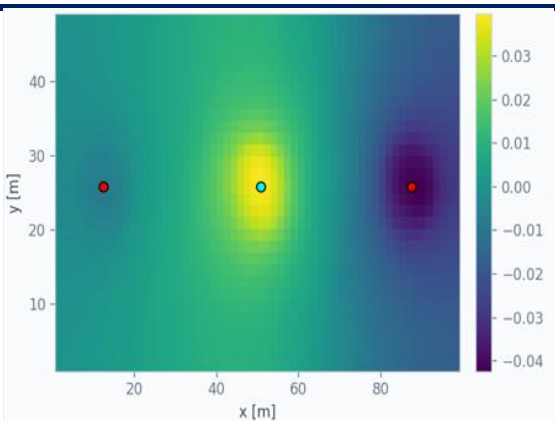
The calculation results were performed using the Python programming language. Python programming offers significant advantages for modeling, simulation, and optimization of ISL processes. Its flexibility, extensive scientific libraries, and ease of integration make Python a powerful tool for addressing the complex hydrogeological and geochemical challenges associated with ISL operations. One major advantage of Python is its strong support for numerical modeling and data analysis. Libraries such as NumPy, SciPy, and Pandas enable efficient handling of large datasets related to pressure, permeability, flow rates, and solute concentration. These capabilities are essential for simulating fluid flow and mass transport within ore-bearing formations. When solving the problem, the dimensions of the sphere were taken as follows: $0 < x < 100$ (m), $0 < y < 50$ (m), z-varies depending on the ore reserve. **Fig. 1–4** show the distribution of acid in the ore-bearing plain within the mine. Production well is located in the center, and two injection wells are located at equal distances on both sides of it. Over time, the pressure increases and the dynamics of acid distribution along the medium expands. The results below reflected the spread of pressure in the medium in various situations. The results were obtained when the diameters of the productions wells were 10cm and 20cm, respectively, and the diameter of the injection well was 25cm. All results were generated within 15 days of the process.



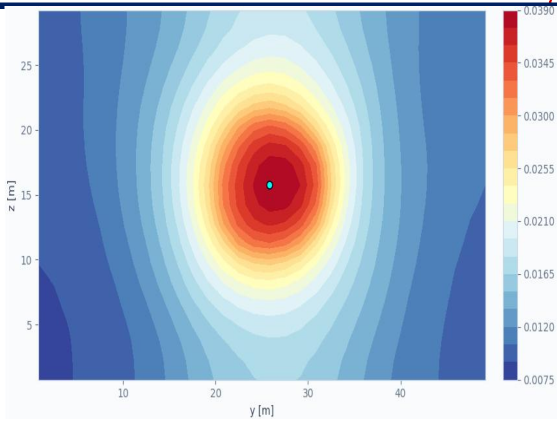
a



b



c



d

Figure-1

In figure-1, graph a shows the pressure distribution from the center of the sphere in contour form. Graphs b and c illustrate different views of the pressure distribution in the sphere. Graph d shows the projection of the pressure distribution on the *Oxz* plane (x=50).

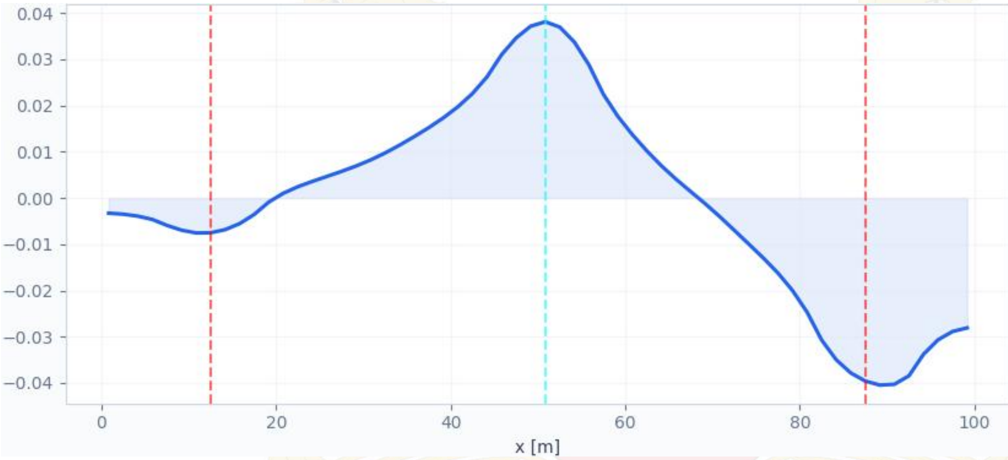


Figure-2

Figure 2 shows the variation of the pressure function with respect to the variable x. Due to the pressure distribution through the source well, the pressure is high in the center and low around the production wells.

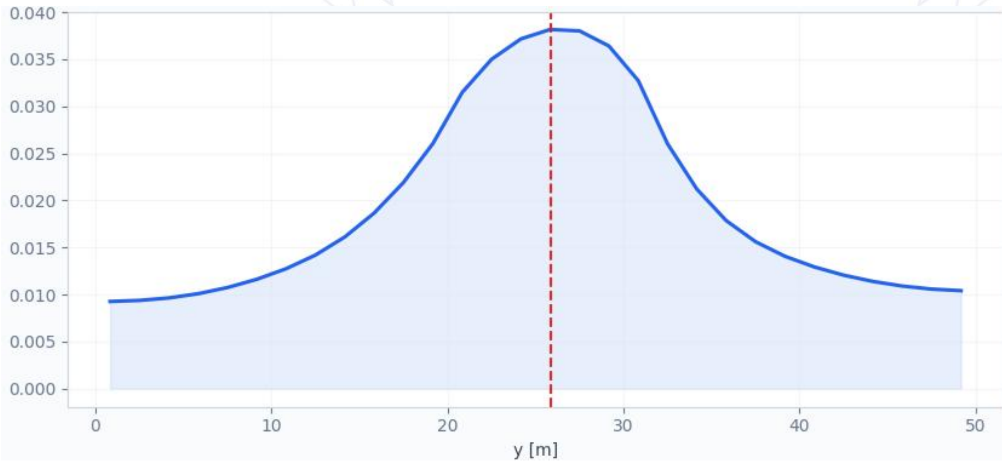


Figure-3

Figure 3 shows the variation of the pressure function with respect to the variable y. The receiving wells are not included in the image because the well locations are parallel to the Ox axis.

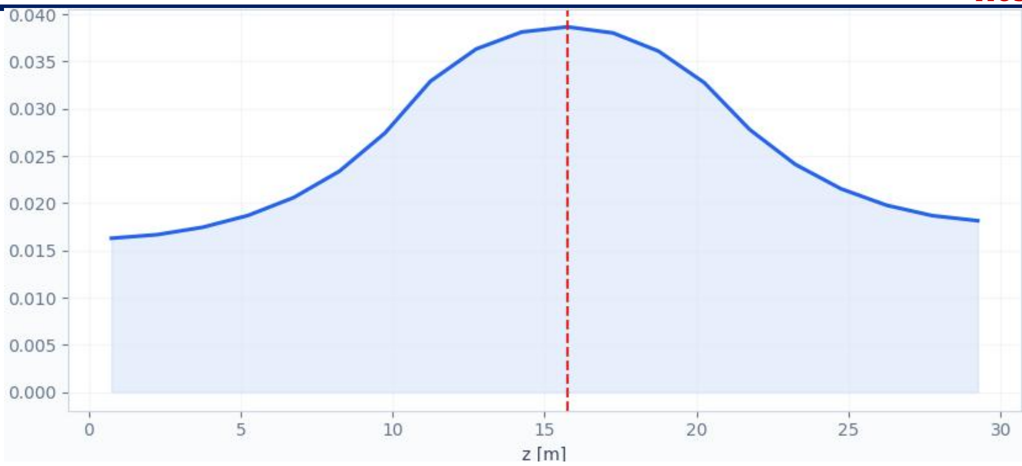


Figure-4

Figure 3 shows the variation of the pressure function with respect to the z variable. Since the wells are located 15 meters below the origin, the pressure reaches its highest value at this height.

Conclusion

The solution to this problem is carried out through the combined action of filtration, diffusion and kinetic processes. The movement of a liquid is determined by the volume or surface forces acting on it. The passage of substances in a moving liquid occurs through two completely different mechanisms. First, when there is a difference in concentrations in the liquid, molecular diffusion occurs, and secondly, a substance dissolved in a liquid is absorbed by the second during its movement. The combination of both processes is called convective diffusion of substances in a liquid. It should be noted that in the process of ISL, firstly, as a result of a change in pressure in a porous medium, and secondly, as a result of the dissolution of substance particles during acid cleaning of an ore deposit, the main hydrodynamic parameters of the object of study change.

Future research will focus on refining this model to include dynamic factors such as chemical interactions, multiphase flow motion, and integration of real-time field data. Further advances in computational algorithms and software will also increase the accuracy and applicability of simulations, leading to more sustainable and efficient mining practices.

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