VECTOR-VALUED FUNCTIONS AND THEIR APPLICATION IN ENGINEERING PROBLEMS

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ABSTRACT

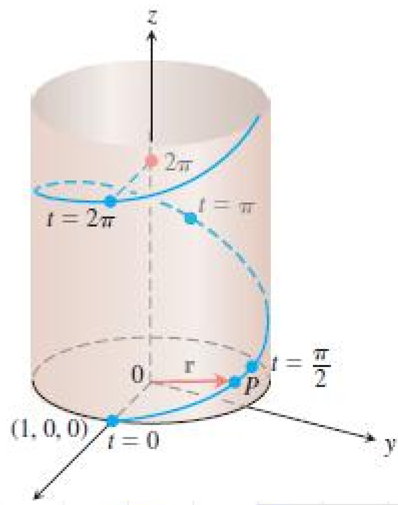
Vector-valued functions are widely used in various branches of engineering. These functions are applied in mechanics to describe trajectories, in electrical engineering to analyze fields, and in civil engineering to evaluate forces..

Vector-valued functions play an important role in vector analysis and are extensively applied across different engineering disciplines. These functions take scalar or vector arguments and produce a vector as a result. In mechanics, electrical engineering, civil engineering, and other fields, vector-valued functions serve as essential tools for modeling various phenomena.

A vector-valued function is defined as a function that maps a scalar variable t to a vector $\vec{r}(t)$:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k},$$

where $x(t)$, $y(t)$, $z(t)$ are scalar components, and $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors in three-dimensional space.



For example, the function

$\vec{r}(t) = cos(t)\vec{i} + sin(t)\vec{j} + t\vec{k}$ describes a helical trajectory. Its derivative

$$\vec{r}'^{(t)} =- sin(t)\vec{i} + cos(t)\vec{j} + \vec{k}$$

represents the velocity vector.

Vector-valued functions are used in the following fields of engineering:

1. Mechanics: Modeling Motion.

In describing motion, vector-valued functions are used to determine an object's position, velocity, and acceleration.

Example: Parabolic motion.

The trajectory of a projectile is given by $\vec{r}(t) = 50t\vec{i} + (20t - 4.9t^2)\vec{j}$, where t is

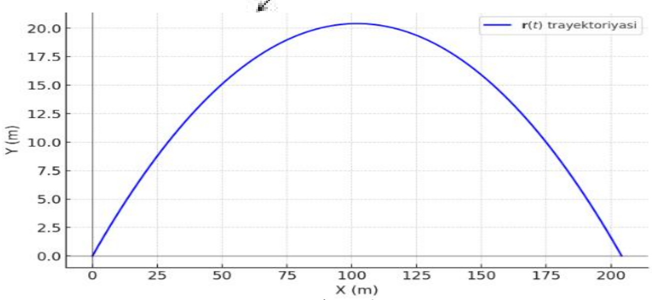


Figure 2.

time in seconds, 50 is the horizontal velocity (m/s), and 4.9 is half the gravitational acceleration (m/s^2).

- Velocity: $\vec{r}(t) = 50\vec{i} + (20 - 9.8t)\vec{j}$
- At the highest point: $v_y = 0$: $20 - 9.8t = 0 \rightarrow t \approx 2.04\ s$.

Height: $y(2.04) = 20 \cdot 2.04 - 4.9 \cdot (2.04)^2 \approx 20.4$m.

The motion trajectory of the projectile is shown in Figure 2. This example is used in engineering to calculate distance.

We use the Python program to plot the graph of this function:

```
# Y = 0 bo'lgan t ni to'g'ri topish
t_roots = fsolve(trajectory_eq, [0, 5])  # Ikki ildizni topish
t_max = max(t_roots)  # Nolga teng bo'lmagan eng katta ildizni olish
# Yangilangan vaqt massivini yaratish
t = np.linspace(0, t_max, 100)
# Yangilangan X va Y qiymatlari
x_values = 50 * t
y_values = 20 * t - 4.9 * t**2
# Grafikni chizish
plt.figure(figsize=(8, 5))
plt.plot(x_values, y_values, 'b-', label="Harakat trayektoriyasi")  # Trayektoriya
# Vektorlarni chizish
for i in range(0, len(t), 10):
   dx = 50  # X o'qi bo'ylab tezlik
   dy = 20 - 9.8 * t[i]  # Y o'qi bo'ylab tezlik
   plt.quiver(x_values[i], y_values[i], dx, dy, angles='xy', scale_units='xy', scale=200, color='r')
# Grafik bezaklari
plt.xlabel('X koordinatasi (m)')
plt.ylabel('Y koordinatasi (m)')
plt.title('Vektor qiymatli funksiya grafigi (Ox o‘qini kesguncha)')
plt.axhline(0, color='black', linewidth=1)  # Ox o‘qi
plt.axvline(0, color='black', linewidth=1, linestyle='--')  # Oy o‘qi
plt.grid()
plt.legend()
plt.show()
```

2. Electromagnetic Fields.

Vector-valued functions describe variations of electric and magnetic fields.

Example: Electromagnetic wave. The electric field is defined as

$$E(t) = E_0 cos(\omega t - kz)\vec{i},$$

where E_0 – is the amplitude, ω – is the frequency, and k – is the wave number, z – distance. The derivative

$$\frac{dE}{dt} = -E_0 \omega sin(\omega t - kz)\vec{i},$$

is important for analyzing signal propagation in wireless communication systems.

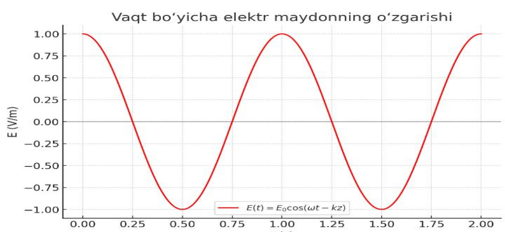


Figure 3.

When $E = 1$, $\omega = 2\pi$, $k = 1$, the graph of the change in the electric field over time has the following form (Fig. 3):

3. Structural Engineering.

Vector-valued functions are used to analyze force distribution and structural stability.

Example: Load distribution.

The load acting on a building foundation is expressed as $P(t) = (2000 - 10t)\vec{j}$, where t is time (in years) and the load decreases due to erosion. This is important for assessing long-term stability in civil engineering.

Vector-valued functions are necessary for the precise modeling of real processes in engineering. Their role in the analysis of trajectories in mechanics, fields in electrical engineering, and forces in construction is invaluable. The above examples show that vector-valued functions are not only a theoretical basis, but also an important tool in practical design. Therefore, in-depth study of this field is of great importance for engineering students and specialists.

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